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Determination of the Fracture Toughness Parameters of Quasi-Brittle Materials Using Cylindrical Samples

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Abstract

A method of determination of the critical stress intensity factor for brittle and quasi-brittle materials (ceramics, cement based materials) is proposed. In the approach proposed the process of sample fracture develops from the vertex of a V-shaped notch, in contrast to standard methods, where the fracture process starts from a crack tip. For the compressed cylindrical sample with central diamond-shaped hole, the stress intensity factors were calculated for an arbitrary vertex angle. The critical values of SIF were obtained by considering a deformation fracture criterion based on the Dugdale model of failure with the assumption that the fracture process zone starts from the notch vertex. The approach presented can be applied to the analysis of the fracture process in the vicinity of stress concentrators such as sharp and round notches formed in the sample.

Keywords: Fracture mechanics, Stress intensity factor, V-notch, Stress concentration, Fracture testing

OZNACZANIE PARAMETRÓW ODPORNOŚCI NA PĘKANIE MATERIAŁÓW QUASI-KRUCHYCH NA PRÓBKACH CYLINDRYCZNYCH

Zaproponowano metodę oznaczania współczynnika krytycznej intensywności naprężeń w przypadku materiałów kruchych i quasi-kruchych (ceramika, materiały na bazie cementu). W zaproponowanym podejściu proces pękania próbki rozwija się od wierzchołka karbu w kształcie V w przeciwieństwie do metod standardowych, w których proces pękania rozpoczyna się na wierzchołku pęknięcia. W przypadku ściskanej próbki cylindrycznej zawierającej otwór w kształcie diamentu współczynniki intensywności naprężeń (SIF) zostały obliczone dla arbitralnie przyjętego kąta wierzchołkowego. Wartości krytyczne SIF uzyskano przyjmując kryterium pękania dla deformacji opartej na modelu Dugdale'a przy założeniu, że strefa pękania rozpoczyna się od wierzchołka karbu. Zaprezentowane podejście można zastosować do analizy procesu pękania w sąsiedztwie koncentratorów naprężeń takich jak ostre i zaokrąglone karby utworzone w próbce.

Słowa kluczowe: mechanika pękania, współczynnik intensywności naprężeń, karb w kształcie V, koncentracja naprężeń, badanie pękania

1. Introduction

Ceramics and ceramic composites are materials having high strength characteristics but quite low crack resistance properties at the same time. The crack resistance is critical both for structural elements operating under extreme loads and when brittle fracture is intolerable under arbitrary loads as well. The use of the methods of linear fracture mechanics to analyze fracture processes in structural elements made of cementitious materials has over a half-century tradition. Works [1, 2] contain reviews of achievements in this domain. Fracture toughness investigations of ceramics (zirconia, alumina and silicon) started only in the late 1980s [3].

A basic fracture toughness parameter of materials is the critical stress intensity factor measured experimentally on samples with cracks. In the case of metals the procedure of determining this parameter is standardized and universally applied. The process of fracture starts, in this instance, from a pre-existing sharp crack tip initiated by fatigue loading. For brittle materials, such as concrete, ceramics or rocks, the process of obtaining the initial crack is more difficult to control.

The rough estimation of the initial fatigue crack length can also be the cause of inaccuracy. Usually, for brittle materials, such as concrete, this initial crack is formed at the casting stage of the sample [4, 5] or by a diamond saw cut [2]. The U-shaped notches obtained in this manner are 2-3 mm in width and have a similar notch tip rounding diameter. Consequently, all other dimensions of the specimen must be relatively large, which raises the costs of research. Also, the influence of the notch radius on the fracture toughness measurements must be taken into account [6, 7]. The latest recommendations RILEM [2] allow determination of fracture parameters on compact specimens such as cubes and cylinders (which are used commonly for monitoring the quality of concrete mixtures) with the wedge splitting test.

In this paper a method of determination of the critical stress intensity factor on cylindrical specimens with a centrally situated diamond-shaped opening is proposed. The fracture process starts from the rounded V-notch vertex. Using the deformation fracture criterion [8] based on the Dugdale model [9] and the value of the failure load, the critical stress intensity factor is calculated.

The choice of the shape of concentrator at the form of the diamond opening is related to the study category of the paper and the wish of comparisons of calculated stress concentration factors with those already published [10, 11], for particular values of the V-notch opening angle. The approach presented can be successfully applied to the analysis of the fracture process in the vicinity of any symmetrical stress concentrators, including sharp and rounded V-notches weakening the section of the sample.

2. Relationship between stress intensity and stress concentration factors for sharp and rounded notches

The value of stress in the apex of a notch with opening angle 2β rounded with a small radius of curvature ρ, can be defined by formula:

$$\sigma_{max} = \frac{K_I^V}{\sqrt{2\pi}} R_I \rho^{-\lambda}, \qquad (1)$$

where K_I^V is the stress intensity factor for the corresponding sharp notch, and R_I is called a stress rounding factor [12]. Bottom indexes I bear upon the type of the stress field caused by tensile loads in the vicinity of notch tip. The exponent of singularity λ is taken as the smallest positive root of the characteristic equation:

$$(1-\lambda)\sin 2\alpha + \sin(2\alpha(1-\lambda)) = 0, \quad \alpha = \pi - \beta.$$

The values of λ can be estimated from the following function [13]:

$$\lambda_I \approx 1.247\cos\beta - 1.312\cos^2\beta + 0.8532\cos^3\beta - 0.2882\cos^4\beta.$$

The maximum absolute error is below 0.001.

The following approximate formula was proposed in [14] for determining the value of stress rounding factor R_I. The formula concerns notches rounded with a circular arc and makes the estimation of R_I possible with the error not exceeding 0.1 % for $\beta < 165^o$:

$$R_I = \frac{1 + 28.75\gamma + 98.04\gamma^2 - 102.1\gamma^3 + 47.4\gamma^4 - 8.465\gamma^5}{1 + 20.71\gamma}.$$

The formulas in the form (1) were published by many authors, *e.g.* Creager and Paris [15], Neuber [16] and others [12, 17], who calculated the values of stress rounding factor R_I or its analogues. For the purpose of solution of the problem by means of analytical or numerical methods, the shape of the infinite wedge was mapped using different smooth curves (parabolic, hyperbolic) with variable radius of curvature in the vicinity of the notch apex. In [14] and [18] it was proved that the stress concentration at the apex of a rounded V-notch under a symmetrical stress state depends on the notch geometry in the vicinity of the notch apex. Thus, an explicit identification of the stress rounding factor values is possible only for a precisely defined notch shape. The correctness of this approach was documented in [13, 14, 18].

3. Stress concentration in the apex of a diamond-shaped oval hole in compressed cylindrical sample

The geometry analyzed is shown in Fig. 1. The corners of the opening are rounded with a circular arc of radius ρ. The relation of the apex curvature radius to the half of the projection of the diamond-shaped oval length on the x-axis is defined by the parameter $\varepsilon = \rho/l$. The relation of the hole length to the radius of the sample R was marked as $\gamma = l/R$. It was assumed that the hole edge (the smooth contour L) is free of external load. The sample is subjected to the balanced forces P acting along the diameter of the cylinder. These forces cause a tensile stress concentration in the apex A of the hole. For the required accuracy of the determination of the value of stress at the edge of the analyzed area, the calculation was carried out using the method of singular integral equations [19].

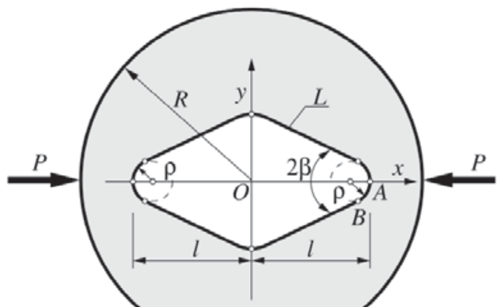

Fig. 1. Diamond-shaped hole in the circular domain.

The complex stress potentials [20] of the boundary value problem were written in the form:

$$\Phi^*(z) = \Phi_0(z) + \Phi(z), \qquad \Psi^*(z) = \Psi_0(z) + \Psi(z), \qquad (2)$$

where potentials:

$$\Phi_0(z) = -\frac{\gamma}{2\pi}\cdot\frac{P}{l}\cdot\frac{R^2 + z^2}{R^2 - z^2},$$

$$\Psi_0(z) = \frac{2\gamma}{\pi}\cdot\frac{P}{l}\cdot\frac{R^4}{(R^2 - z^2)^2}$$

describe the homogeneous field of stress in compressed circular area, and functions $\Phi(z)$ and $\Psi(z)$ define the stress disturbance due to the presence of the hole L. The boundary condition on the notch contour can be described as follows:

$$\sigma_n + i\tau_{ns} = p(t) = -\left\{\Phi_0(t) + \overline{\Phi_0(t)} + \frac{\overline{dt}}{dt}\left[t\overline{\Phi'_0(t)} + \overline{\Psi_0(t)}\right]\right\}, \quad t \in L.. \qquad (3)$$

The integral solution of the boundary problem was presented in the form [19]:

$$\Phi(z) = \frac{1}{\pi}\int_L \left[f_1(t,z)g'(t)dt + f_2(t,z)\overline{g'(t)dt}\right],$$

$$\Psi(z) = \frac{1}{\pi}\int_L \left[h_1(t,z)g'(t)dt + h_2(t,z)\overline{g'(t)dt}\right],$$

where

$$f_1(t,z)=\frac{1}{2}\left[\frac{1}{t-z}+\frac{\bar{t}}{z\bar{t}-R^2}\right],$$

$$f_2(t,z)=\frac{z\left(t\bar{t}-R^2\right)\left(z\bar{t}-2R^2\right)}{2R^2\left(z\bar{t}-R^2\right)^2},$$

$$h_1(t,z)=\frac{1}{2}\left[-\frac{\bar{t}}{(t-z)^2}+\frac{\bar{t}^3}{\left(z\bar{t}-R^2\right)^2}\right],$$

$$h_2(t,z)=\frac{1}{2}\left[\frac{1}{t-z}+\frac{\bar{t}\left[4R^4-3R^2\bar{t}(z+t)+z\bar{t}^{-2}(z+t)\right]}{\left(z\bar{t}-R^2\right)^3}\right].$$

Satisfying the boundary condition (3) the following singular integral equation was obtained:

$$\frac{1}{\pi}\int_L\left[K(t,t')g'(t)dt+L(t,t')\overline{g'(t)dt}\right]=p(t'),\ t'\in L \tag{4}$$

the kernels of which can be expressed by formulas:

$$K(t,t')=f_1(t,t')+\overline{f_2(t,t')}+\frac{\overline{dt'}}{dt}\left[t'\overline{g_2(t,t')}+\overline{h_2(t,t')}\right],$$

$$L(t,t')=f_2(t,t')+\overline{f_1(t,t')}+\frac{\overline{dt'}}{dt}\left[t'\overline{g_1(t,t')}+\overline{h_1(t,t')}\right],$$

where $g_\alpha(t,t')=f_\alpha{}'(t,t')$, α = 1.2.

The equation describing the contour L was written in the parametric form $t = l\omega(\xi)$, (where $0 \le \xi \le 2\pi$). Through variable changes the integral equation (4) can be written in canonical form:

$$\frac{1}{\pi}\int_0^{2\pi}\left[M(\xi,\eta)u(\xi)+N(\xi,\eta)\overline{u(\xi)}\right]d\xi=p(\eta),\quad 0\le\eta\le 2\pi,$$

where

$$M(\xi,\eta)=lK(\omega(\xi),\omega(\eta)),\qquad N(\xi,\eta)=lL(\omega(\xi),\omega(\eta)),$$
$$u(\xi)=g'(\omega(\xi))\omega'(\xi),\quad p(\eta)=p(\omega(\eta)).$$

The unknown 2π-periodic continuous function $u(\xi)$ is quasi-singular at the oval apexes, which prevent the obtainment of the desired accuracy of solution for small values of radius ρ. Recently, for the purpose of improving the accuracy of solution of this type of equation, various non-linear transformations are used. In the case considered the following variable change was applied [18]:

$$\xi=G(\tau)=\tau-(1/4)\sin 4\tau,\quad 0\le\tau\le 2\pi;$$
$$\eta=G(\theta),\quad 0\le\theta\le 2\pi.$$

The function $G(\tau)$ provides one-to-one mapping between the intervals $\tau\in[0,2\pi]$ and $\xi\in[0,2\pi]$. Finally the following integral equation was obtained:

$$\frac{1}{\pi}\int_0^{2\pi}\left[M(\xi,\eta)u^*(\tau)+N(\xi,\eta)\overline{u^*(\tau)}\right]G'(\tau)d\tau=p^*(\theta), \tag{5}$$

for $0\le\theta\le 2\pi$ and $u^*(\tau)=u(G(\tau))$, $p^*(\theta)=p(G(\tau))$.

A discreet analogue of the integral equation is the algebraic system of linear equations [19]:

$$\frac{1}{2n}\sum_{k=1}^{4n}\left[M(\xi_k,\eta_m)u^*(\tau_k)+N(\xi_k,\eta_m)\overline{u^*(\tau_k)}\right]G'(\tau_k)=p^*(\theta_m),$$

where

$$\xi_k=G(\tau_k),\qquad \tau_k=\frac{\pi(2k-1)}{4n},$$
$$\eta_m=G(\theta_m),\qquad \theta_m=\frac{2\pi(m-1)}{4n},\qquad k,m=\overline{1,4n}.$$

For reasons of symmetry about the Ox- and Oy-axes, the order of the system of algebraic equations can be reduced four times. Considering the following conditions:

$$u^*(2\pi-\tau)=\overline{u^*(\tau)},\qquad u^*(\pi-\tau)=-\overline{u^*(\tau)},$$

the system of n complex algebraic equations was obtained:

$$\begin{cases}\dfrac{1}{n}\mathrm{Re}\sum_{k=1}^{n}\left[M^*(\xi_k,\eta_m)u^*(\tau_k)\right]G'(\tau_k)=p^*(\theta_m), & m=1,\\[2ex] \dfrac{1}{2n}\sum_{k=1}^{n}\left[M^*(\xi_k,\eta_m)u^*(\tau_k)+N^*(\xi_k,\eta_m)\overline{u^*(\tau_k)}\right]G'(\tau_k) & \\ \qquad =p^*(\theta_m), & m=\overline{2,\dots,n},\\[1ex] \dfrac{1}{n}\mathrm{Re}\sum_{k=1}^{n}\left[M^*(\xi_k,\eta_m)u^*(\tau_k)\right]G'(\tau_k)=p^*(\theta_m), & m=n+1,\end{cases}$$

for n unknown values of function $u^*(\tau_k)$, $k=\overline{1,\dots,n}$. Here:

$$M^*(\xi_k,\eta_m)=M(\xi_k,\eta_m)-N(\pi-\xi_k,\eta_m)-$$
$$-M(\pi+\xi_k,\eta_m)+N(2\pi-\xi_k,\eta_m),$$
$$N^*(\xi_k,\eta_m)=N(\xi_k,\eta_m)-M(\pi-\xi_k,\eta_m)-$$
$$-N(\pi+\xi_k,\eta_m)+M(2\pi-\xi_k,\eta_m).$$

Solving the system of algebraic equations, we find the complex potentials of stresses (2), which define the stress and deformation state in the whole elastic plate. The normal stresses σ_s in points lying on the edge of the hole are calculated directly from the determined function $u^*(\tau)$ [21]. The stress concentration factor in the apex A (Fig. 1) is calculated according to the formula:

$$k_A=\frac{(\sigma_s)_{\max}}{P/l}=-4\,\mathrm{Im}\left[u^*(0)/\omega'(0)\right],$$

whereas the value $u^*(0)$ is determined using the Lagrange interpolation.

Calculations were carried out for two independent parameters – γ of value changing from 0.05 to 0.9875 and the notch opening angle 2β from 0 to $\pi/4$. The stress intensity factor K_I^V in the sharp corner of the diamond-shaped hole was calculated according to the following formula [14]:

$$K_I^V=\frac{\sqrt{2\pi}}{R_l}l^{\lambda}\lim_{\rho\to 0}\varepsilon^{\lambda}(\sigma_s)_{\max} \tag{6}$$

Satisfactory accuracy of the limit transition (6) has been reached for values of ε from 0.00001 to 0.0001 depending on the value of angle β. The variation of the dimensionless stress intensity factor

$$F_I^V=K_I^V/\left(P^{\lambda-1}\sqrt{\pi}\right) \tag{7}$$

as a function of the parameter γ is presented in Fig. 2. The obtained value of the dimensionless stress intensity factor for 2β = 0 agrees well with data given in the literature for a disk weakened by the central crack [11]. The differences

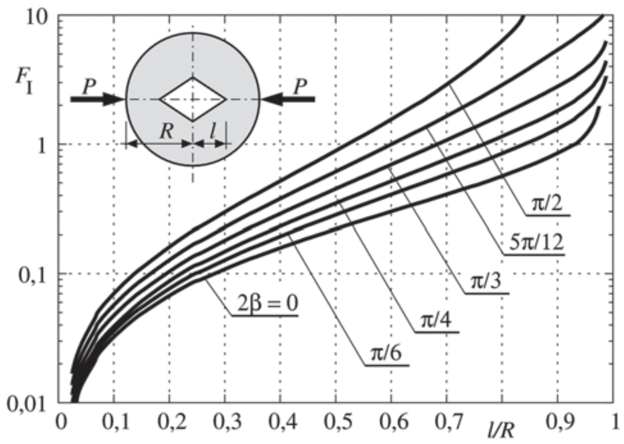

Fig. 2. Dimensionless stress intensity factor F_I^V as a function of parameter γ = l/R.

in results do not exceed 0.1 %.

For several chosen values of the parameter γ = {0.2, 0.3, 0.4, 0.5}, the approximating formula has been obtained for the dimensionless stress intensity at the notch apex. In the range of $0 \le \gamma \le \pi/4$, the function can be approximated by means of the polynomial of 3-degree, with the error not exceeding 0.5 %:

$$F_I^V \approx a + bx + cx^2 + dx^3, \qquad x = \mathrm{tg}\beta. \tag{8}$$

The values of coefficients in formula (8) were given in Table 1.

Table 1. Coefficients of the polynomial (8) approximating the value of the stress intensity factor.

γ = l/R	a	b	c	d
0.2	0.06748	0.00226	0.1233	-0.0321
0.3	0.10843	0.01522	0.2074	-0.0335
0.4	0.15828	0.04529	0.3135	0.0000
0.5	0.22079	0.09856	0.4471	0.1195

4. Critical stress intensity factor

Carrying out the research on the samples described and using the formula (7) make it possible to determine experimentally the function of the critical stress intensity factor $K_{Ic}^V(\beta)$, which gives the basis for stress fracture criteria of construction elements [22]. For the opening angle of the notch β = 0, *i.e.* for the case of the crack, it is possible to determine the critical stress concentration factor K_c [4, 23]. The value of material constant K_c can also be determined using the deformation criterion described in [8], as a function of the crack opening displacement $\delta_I(\beta)$ in the apex of the V notch. For brittle materials for which the range of the plastic zone is small in comparison to the notch depth, the values of the function $\delta_I(\beta)$ depend only on the material constants. Taking into account results obtained by [8] the critical notch stress intensity factor can be described by dependence:

$$K_{Ic}^V(\beta) = (\sigma_Y)^{1-2\lambda}\left[\frac{K_c^2}{\bar{\delta}_I(\beta)}\right]^{\lambda}, \tag{9}$$

where stress σ_Y complies with the material yield point and $\bar{\delta}_I(\beta)$ is the dimensionless value of the crack opening displacement $\delta_I(\beta)$. Using the formulas (7) and (9) the critical stress intensity factor is calculated according to formula:

$$K_c = \left[\sqrt{\pi}F_I^V\bar{\delta}_I(\beta)\frac{P_c l^{\lambda-1}}{(\sigma_Y)^{1-2\lambda}}\right]^{1/(2\lambda)},$$

where P_c is the force causing sample destruction.

One can calculate dimensionless crack opening displacement at the sharp (ρ = 0) notch vertex using the simple approximated formula [8]:

$$\bar{\delta}_I(\beta) \approx \frac{4}{\pi^2}\frac{\lambda}{1-\lambda}\left[\frac{\Gamma(1-\lambda)\Gamma(0.5)}{2\Gamma(1.5-\lambda)}\right]^{1/\lambda}.$$

For samples with rounded notches (ρ > 0), a more accurate relation could be found in [13].

5. Conclusions

The method presented in the paper can be applied to precise estimation of the critical stress intensity factor of brittle and quasi-brittle materials using samples of different shapes with stress concentrators in the form of sharp or round notches.

The use of deformation fracture criterion, containing only standard material constants allows determination of the material critical stress intensity factor from experiments carried out on samples with notches, which greatly simplifies the preparation of samples – formed or cut out from the structure.

The approach presented can also be applied to the analysis of fracture process in the vicinity of stress concentrators situated in any structural component.

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